SETON HALL UNIVERSITY

TWENTY-SEVENTH ANNUAL

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MATHEMATICS COMPETITION

1. Find the real number x for which the sum of x, 2 more than three times x, and 1 less than seven times x is 23.

2. The wind pressure on a sail varies jointly as the area of the sail and the square of the velocity of the wind. If the pressure is 1 pound per square foot when the velocity of the wind is 15 miles per hour, find the pressure (in pounds per square foot) for a wind of 4 times this velocity.

3. A jeweler has 7 rings, each weighing 16 g, made of an alloy of 15% silver and 85% gold. He plans to melt down the rings and add enough silver to reduce the gold content to 70%. Find the number of grams of silver he should add.

4. A cube K has volume R^3 (where R is a positive real number). Eight spheres are drawn; each sphere has center at a vertex of cube K and radius half the length of an edge of cube K. Find (in terms of R) the area of the surface S interior to cube K formed by the eight spheres. (Note that the total surface area of a sphere is 4π times the square of the length of its radius.)

5. A bakery has 2 large ovens and 3 small ones. To fill a certain order would take either of the large ovens 5 hours and would take any one of the small ovens 7 hours. If all five ovens are used, how long will it take to fill the order?

6. Find the largest negative integer x such that $|3x+7| > \frac{25}{3}$.

7. An auto was driven 200 miles in 5 hours. Part of the distance was in a rural area at an average speed of 60 mph, the next part of the distance was in the suburbs at an average rate of 45 mph, and the rest was in a city at an average speed of 20 mph. If the distance driven in the city was 1/5 of the total distance, how many hours of the drive were in the rural area?

8. Find the coordinates (x, y) of all points on a coordinate plane for which (1) the distance from (x, y) to $(10, 0)$ is twice the distance from (x, y) to $(4, 0)$, and (2) the square of the distance from (x, y) to $(0, 0)$ is twice the square of the distance from (x, y) to $(6, 0)$.

9. Le B be an integer larger than 1, let V be a real number larger than 1, and let $C = V + 1$. Write the expression $B^{V\log_B C} + \left(\log_B B^V\right)^C + \left(B^{\log_B C} + \log_B B^V\right)^{V+C}$ in simplest form in terms of V.

10. Simplify (where w is a positive real number and $2w^2 < 1$):

$$\frac{w^3 - \dfrac{w^4}{\sqrt{1-w^2}}}{1-2w\sqrt{1-w^2}} \ \square \ \sqrt{\frac{1}{w^2}-1} \ \square \ \frac{w-\sqrt{1-w^2}}{w}.$$

11. Two equal circles on a plane, each of radius length r (where r is a positive real number), are tangent externally at point T. T is the center of a coplanar circle C of radius length $2r$. A chord RS of circle C is drawn tangent to both smaller circles at points P and Q, respectively, forming a segment D of circle C enclosed by chord RS and its minor arc RS. A point inside circle C is chosen at random. Find the probability that this point lies in segment D.

12. Find the term (in simplified form) in the binomial expansion of $\left(\frac{3}{2}x^6y^{-2} - \frac{1}{9}x^{-1}y^3\right)^8$ in which the sum of the exponents of x and y added to the product of the exponents of x and y is -1.

13. Let n denote the number of sides of a regular polygon (where n is a positive integer larger than 2) and let $d(n)$ denote the degree measure of each angle in a regular polygon of n sides. Consider the values of n for which $d(n)$ is an integer. Find the least possible sum of these n which are terms in an arithmetic progression having at least five terms; find the greatest possible sum of these n which are terms in a geometric progression having at least four terms.

14. Solve the given trigonometric equation for non-negative values of x less than 2π:

$$4\sin^2 x - 2\sin x\cos^2 x(2\sin x + 3) + 3\sin x - 1 = 0.$$

15. Triangle ABC is a right triangle with side AB perpendicular to side AC. Line segment AD meets side BC at point D and AD is perpendicular to BC at D. The length of line segment AD is 24 feet and is 12/5 the length of line segment BD. Find the perimeter and the area of triangle ABC.

16. A deck of 24 cards includes 6 red, 6 green, 6 yellow, and 6 blue cards, numbered from 1 to 6, respectively. If five cards are drawn at random, find the probability that four 2's and one odd number are drawn (1) with replacement and (2) without replacement. (Express answers as rational numbers in simplest form)