1) The lengths of a diagonal of a cube *C* and a diagonal of a face of cube *C* differ by 4 inches. Find the length of a side of cube *C*.

ANS: $4(\sqrt{3}+\sqrt{2})$ inches

2) Find *B*, if *B* is a positive integer such that $(100_3)(12_4)+(253_6)=237_B$.

ANS: 8

3) Tom runs one-fourth of the distance from his home to the bus stop at the rate of 7 miles per hour, one-eighth of the distance at the rate of 6 miles per hour, and the remaining five-eighths of the distance at the rate of 5 miles per hour. At what average rate did Tom travel from his home to the bus stop?

ANS: $\frac{336}{61}=5\frac{31}{61}$ mph

4) Let *B* be a real number, *B* > 2. If $\log_B\left(B^t\cdot B^{2t}\cdot B^{3t}\cdot B^{4t}\cdot B^{5t}\cdot B^{6t}\cdot B^{7t}\cdot B^{8t}\cdot B^{9t}\cdot B^{10t}\right)=B^{\log_B 15+\log_B 11}$, find the real number *t*.

ANS: 3

5) The vertices of a triangle are the points *A* = (10, 0), *B* = (0, 24), and *C* = (18, 0) (on the *xy*-plane). Find the *x*-intercept of the (coplanar) line which bisects angle *ABC*.

ANS: $\left(\frac{96}{7},0\right)=\left(13\frac{5}{7},0\right)$

6) The numerator of a positive fraction exceeds three times its denominator by 2. If the numerator is decreased by 32 and the denominator is increased by 32, the resulting fraction will equal the reciprocal of the given fraction. Find the denominator of the original fraction.

ANS: 15

7) Find all real numbers *x* for which $\sqrt[4]{9x^2+12x+4}+\sqrt[3]{27x^3-54x^2+36x-8}+\sqrt{3x+2}-6x=0$.

ANS: 2/3, -2/3

8) Joan can distribute 4 (distinguishable) blocks (1 red, 1 green, 1 blue, 1 yellow) among 10 children in *D* ways. She can distribute 4 (indistinguishable) red blocks among 10 children in *N* ways. Find the value of $D - N$. (Note that in either case, a child can get more than one block).

ANS: 9285

9) Points *A* and *B* lie on line ℓ in 3-dimensional space and the distance between *A* and *B* is 8 feet. *T* and *S* are two points on ℓ each 3 times as far from *A* as from *B*. Points in space which are 8 feet from *A* or 8 feet from *B* and are on planes through *T* and *S* which are perpendicular to ℓ lie on circles. Find the sum of the areas of these circles.

Ans: 136π ft^2

10) Simplify $\dfrac{\left(\frac{10t}{w-2t}+5\right)\frac{8t^5}{2w^3t+w^3+4w^2t^2+2w^2t+8wt^3+4wt^2}}{\frac{w^3}{w^3-8t^3}-1}$, where *w*, *t* are distinct real numbers, $w\neq 0$, $t\neq 0,-\frac{1}{2}$.

ANS: $\frac{5t^2}{2t+1}$

11) The hyperbola $9x^2-25y^2=225$ and the ellipse $9x^2+16y^2-36x+96y+36=0$ lie on a coordinate plane. Find the shortest distance from the center of the ellipse to the asymptote of the hyperbola having positive slope.

ANS: $\frac{21\sqrt{34}}{34}$

12) $S(n)$ is defined to be the sum of the distinct prime divisors of an integer *n* where *n* is larger than 1. (For example, if *n* = 8800 = $2^5\cdot5^2\cdot11$ then $S(8800)=2+5+11=18$; and *S*(3) = 3.) Find the smallest positive 4-digit integer for which $S(n)$ = 5.

ANS: 1152

13) The lengths of the three sides of triangle *ABC* are $\overline{AB}=42$ feet, $\overline{AC}=18$ feet and $\overline{BC}=30$ feet. Find tan(*A*).

ANS: $\frac{5\sqrt{3}}{11}$

14) Red, green, blue, and white emblems sell for \$12, \$10, \$8, and \$7, respectively. Nancy bought 34 emblems for \$326, with the total cost of green and blue emblems exceeding the total cost of red emblems by \$44. The total number of red and white emblems she bought was 2 less than the total number of green and blue emblems she bought. How many blue emblems did Nancy buy?

ANS: 8

15) Andy can complete a certain task alone in half the number of hours that Bob can complete the task alone and also in half the number of hours that Carl can complete the task alone. Andy worked on the task alone for a few hours, then Bob continued (alone) for 4/5 as long as Andy, and the task was finished by Carl (alone) in 3/2 the time Bob worked on it. The entire task can be completed by all three working on it in 1/2 hour less than it took Carl to finish the task. In how many hours can Andy and Bob working together complete the entire task?

ANS: $\frac{10}{3}=3\frac{1}{3}$ hours

16) Three vertices of a rhombus *ABCD* lie on a circle and the fourth vertex is inside the circle. The perimeter of the rhombus equals the length of a radius of the circle. If the circle has area 100π square inches, find the area of the rhombus.

ANS: $\frac{75\sqrt{7}}{128}$ in^2